

# Primordial black holes, astrophysical systems and the Eddington-Weinberg relation

Salvatore Capozziello<sup>1,2</sup>, Gerardo Cristofano<sup>1,2</sup>, Mariafelicia De Laurentis<sup>1,2</sup>

<sup>1</sup>*Dipartimento di Scienze Fisiche, Università di Napoli "Federico II",*

<sup>2</sup>*INFN Sez. di Napoli, Compl. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126, Napoli, Italy*

(Dated: October 7, 2011)

Starting from a quantization relation for primordial black holes, it is shown that quantum fluctuations can play a fundamental role in determining the effective scales of self-gravitating astrophysical systems. Furthermore the Eddington-Weinberg relation between the current scale of the observed universe to the Planck constant (the natural action unit) is naturally derived. Finally, such an approach allows to recover the current value of the cosmological constant.

PACS numbers: 04.50.+h, 95.36.+x, 98.80.-k

Keywords: cosmology; quantum field theory; large scale structure; cosmological constant

## I. INTRODUCTION

The theoretical understanding of the observed scales, sizes and dimensions of self-gravitating aggregated structures in the universe (stars, galaxies, clusters, etc.) is a long-standing open problem in astrophysics and cosmology. Together with this issue, there have been many attempts to connect such macroscopic features with primordial quantities at the very beginning of the universe history, that is at Planck epoch and beyond. Besides, several decades ago Dirac and Eddington found certain coincidences between large numbers, relating the size and the age of the observed universe to the Compton wavelength of the proton and to the time for traveling the proton size at light speed, respectively [1–5]. Furthermore Dirac strongly believed that such relations are not mere coincidences but hints for some fundamental laws of nature. In general, such relations appear very interesting since they connect macroscopic scales, as for example the present radius of the universe, with the quantum constant  $\hbar$  which can be considered as the natural action unit [6–8]. In other words, one would be induced to think that all macroscopic structures (and the universe itself) have taken origin from quantum phenomena (quantum fluctuations), which were at work at the origin of the universe. In some sense, this is the philosophy underlying the Quantum Cosmology [9]. In the context of large numbers coincidences, also the cosmological constant  $\Lambda$ , today considered the main ingredient for the universe acceleration, can be expressed in terms of the Compton wavelength of the proton, even though its previous estimate by Zeldovich is off of several order of magnitude with respect to the current estimates [21]. Unfortunately, at the moment, there is no non-perturbative field theory capable of giving such coincidences as a self-consistent finding or establishing any definite relation between large and small scales.

In this paper, we propose a straightforward approach to this problem, based essentially on quantum mechanics. Our starting point is the Dirac quantization relation applied to charged primordial black holes [10]. The main result of such a description relies on the scaling properties of quantum relations that can be directly connected

to astrophysical systems and large scale structure. The paper is organized as follows. Sec. II is devoted to the scaling properties of quantization relation for primordial black holes which can be considered as the seeds of large scale structure. In Sec. III, the effective scales of self-gravitating astrophysical structures are derived by using the quantization relation for black holes. A fundamental constant  $a_0$  with the dimensions of an acceleration is derived. In Sec. IV, the Eddington-Weinberg relation is reproduced while, in Sec. V, it is shown how the current value of the cosmological constant  $\Lambda$  is recovered. Conclusions are drawn in Sec. VI.

## II. SCALING HYPOTHESIS FROM BLACK HOLES QUANTIZATION

In a previous paper [10], a quantization relation has been proposed for black holes by adopting the Dirac quantization condition that relates the electric and magnetic charges of a black hole to its mass. It is interesting to stress that such a quantization, derived at Planck scales, can be the seed of scaling relations for all self-gravitating astrophysical systems that we observe now in the universe. Here we explicitly give such a connection, emphasizing the role played by quantum fluctuations at the very early epochs. Let us shortly review the quantization relation derived in [10], starting from the black hole effective potential

$$V_{BH} = e^{2\phi} (Q_e - aQ_m)^2 + e^{-2\phi} Q_m^2, \quad (1)$$

where  $Q_e$  and  $Q_m$  are the electric and magnetic charges of the black hole,  $\phi$  is the dilaton field and " $a$ " the axion field. We recall that this kind of black holes are called *extremal*, that is their event horizons become degenerate. Besides, from Conformal Field Theory, we have

$$V_{eff}^{CFT} = R_c^2 \left( Q_e - \frac{\theta}{2\pi} Q_m \right)^2 + \frac{1}{R_c^2} Q_m^2, \quad (2)$$

where  $R_c$  is the compactification radius of the so-called *Fubini scalar field* [12] and  $\frac{\theta}{2\pi}$  is the *theta parameter* [13].

Comparing Eq. (1) with Eq. (2), we get the following identifications

$$R_c^2 = e^{2\phi}, \quad \frac{\theta}{2\pi} = a. \quad (3)$$

Without loosing generality, we will consider the case  $a = \frac{\theta}{2\pi} = 0$  from now on, that is we will discuss a black hole effective potential in which only the dilaton field is present [11]:

$$V_{BH} = e^{2\phi} Q_e^2 + e^{-2\phi} Q_m^2. \quad (4)$$

Using such a potential and imposing the criticality condition

$$\frac{\partial V_{BH}}{\partial \phi} = 0, \quad (5)$$

one obtains

$$e^{2\phi_H} = \frac{Q_m}{Q_e} = R_H^2 \quad (6)$$

where  $\phi_H$  and  $R_H$  indicate the corresponding values at the horizon of the black hole. Furthermore since such black holes are also extremal, their mass saturates the so-called "Bogomol'nyi-Prasad-Sommerfeld" bound (e.g., see [14])

$$M^2 = e^{2\phi_H} Q_e^2 + e^{-2\phi_H} Q_m^2. \quad (7)$$

Notice that, in the metric approach, Eq.(7) is a consequence of assuming the metric time-time component  $g_{tt} = 0$ . This is an indication of a phase transition, that is charged black holes are forming with mass  $M$  and charges  $Q_e, Q_m$  obeying Eq.(7) (see [10]). Furthermore, substituting Eq.(6) into Eq.(7), we obtain

$$M^2 = 2Q_e Q_m. \quad (8)$$

In order to consider black holes as quantum objects, we assume the Dirac quantization condition

$$2Q_e Q_m = n\hbar c, \quad (9)$$

with  $n$  a positive integer [10]. Substituting into the previous relation and introducing standard units, one gets

$$GM^2 = n\hbar c. \quad (10)$$

For  $n = 1$ , we obtain the lowest mass allowed for a quantum black hole (primordial black hole):

$$M_{BH} = \sqrt{\frac{\hbar c}{G}} = M_{Planck}. \quad (11)$$

It is interesting to notice that, in the quantum relation (10), there is no remnant of the electric and magnetic charges of the object of mass  $M$ , instead its angular momentum  $J = n\hbar$  appears on the right side of the relation

thanks to Dirac quantization. The relation (10) has been proved to be valid for any "self-gravitating" systems [10].

Here we want to study the scaling properties of this quantum relation. To this end, let us recall that the factor  $n$  can be expressed, considering the Compton length  $\lambda$  and the Schwarzschild radius as

$$n_{as} = \frac{R_{as}^{Schw}}{2\lambda_{as}} = \left( \frac{GM_{as}}{c^2} \right) \frac{n_p^{as}}{\lambda_p}, \quad (12)$$

where the label "as" refers to the generic astrophysical structure considered and  $n_p^{as}$  is the number of protons contained in it.

This is the key ingredient of our approach that has to be discussed in details. Eq.(12) points out that the characteristic length of gravitational interaction, the Schwarzschild radius, and the characteristic quantum length for the same structure are related by the granular components of the structure itself, the protons. Such a relation is not arbitrarily assumed but ruled by the Planck length and mass of primordial black holes. Such primordial structures are the first quantized structures emerging from early phase transitions [14] and, according to several inflationary cosmological models, can be considered the seeds for large-scale structure. It is worth stressing that relation (12) strictly depends on the quantum and gravitational interactions and it is not a mere "close packing" where astrophysical and cosmological structures can be built up by the sum of granular elements (see also [8, 10, 17]). In this sense, it is not only the "number" of protons that constitute the structure but mainly their mutual interactions.

With these considerations in mind, Eq.(12) can be recast as

$$n_{as} = const \cdot (n_p^{as})^2, \quad (13)$$

and specializing Eq. (12) to the case of the universe [5, 7], it is not difficult to find that

$$const = \frac{1}{(n_p^{BH})^2}, \quad (14)$$

where  $n_p^{BH}$  indicates the number of protons in the lowest black hole of mass given by Eq.(11). Such a quantity is a constant since, depending on Eq.(11), it is built up only by fundamental constants. The basic quantization relation (10) can be written in a more useful form as

$$GM_{as}^2 = \left( \frac{n_p^{as}}{n_p^{BH}} \right)^2 \hbar c. \quad (15)$$

Finally using for the mass of a given astrophysical structure  $M_{as} = n_p^{as} M_p$ , with  $M_p$  the proton mass, we obtain

$$G(n_p^{as} M_p)^2 = \left( \frac{n_p^{as}}{n_p^{BH}} \right)^2 \hbar c. \quad (16)$$

Furthermore multiplying both members by the factor  $\left(\frac{n_p^{BH}}{n_p^{as}}\right)^2$ , we get

$$G(n_p^{BH}M_p)^2 = \hbar c, \quad (17)$$

that is exactly

$$GM_{Planck}^2 = \hbar c, \quad (18)$$

We then conclude that the quantization relation (10), or (15), scales from the Planck mass to the mass of the different astrophysical structures up to the whole universe.

### III. THE PHYSICAL SIZE OF SELF-GRAVITATING STRUCTURES FROM THE QUANTIZATION RELATION

Let us now show how the physical size of self-gravitating astrophysical systems can naturally arise from the basic quantization relation (10) or, equivalently, from (15). However, in this simplified model, we are assuming that gravity is the leading interaction and protons are the basic constituents. Let us take into account the following relation

$$\frac{GM_{as}}{R_{as}^2} = \frac{GM_u}{R_u^2} = \frac{c^2}{R_u} = a_0. \quad (19)$$

derived considering the Schwarzschild radius and an overall gravitational interaction acting up to the radius of the universe  $R_u$  [8, 17]. The constant  $a_0$  has the physical dimensions of an acceleration and could have a much deeper meaning in connection to the structure formation [20]. It will be discussed in detail elsewhere.

Notice that the previous equation implies

$$\frac{M_{as}}{R_{as}^2} = \frac{M_u}{R_u^2} = const, \quad (20)$$

that we will use below. The numerical factor  $(n_p^{BH})^{-2}$  appearing in Eq. (15), can be written in a more convenient form as

$$\frac{1}{(n_p^{BH})^2} = \left(\frac{M_p}{M_{BH}}\right)^2 \simeq 10^2 \times 10^{-40} \simeq \frac{1}{\sqrt{n_p^u}}, \quad (21)$$

where  $n_p^u \simeq 10^{80}$  indicates the number of protons in the universe. Substituting it back into Eq. (15), we get

$$GM_{as}^2 = 10 \frac{(n_p^{as})^2 \hbar c}{\sqrt{n_p^u}}. \quad (22)$$

Assuming the relation (20), we obtain

$$\sqrt{\frac{n_p^{as}}{n_p^u}} = \sqrt{\frac{n_p^{as}M_p}{n_p^uM_p}} = \sqrt{\frac{M_{as}}{M_u}} = \frac{R_{as}}{R_u}, \quad (23)$$

and, after dividing both members by  $n_p^{as}M_p c^2$ , we have

$$\frac{G(n_p^{as}M_p)}{c^2} = 10 \sqrt{n_p^{as}} \left(\frac{R_{as}}{R_u}\right) \left(\frac{\hbar}{M_p c}\right). \quad (24)$$

Finally dividing left and right sides by  $R_{as}^2$  and making use first of Eq. (20) and then of Eq. (19), we get

$$R_{as} \simeq 10 \sqrt{n_p^{as}} \lambda_p, \quad (25)$$

which, fully reproduces the statistical hypothesis results (see [15, 16]) but with an important extra numerical factor of order 10. Specializing it to the case of the universe, we get

$$R_u \simeq 10 \sqrt{n_p^u} \lambda_p \simeq 10^{28} \text{ cm}, \quad (26)$$

In full agreement with current estimates of the observed radius of the universe. It is interesting to notice that the extra factors 10 or  $10^2$  are perfectly compatible with uncertainties in observations and could be related to the "scatter" between measurements of dark and luminous matter.

An important issue has to be discussed at this point. Clearly we have not used any dark matter hypothesis and the characteristic sizes of astrophysical structures (in particular that of the observed universe) can be reproduced starting from primordial black holes and protons, assumed as granular elements. This is not surprising due to the fact that, in our model, the Compton wavelength (the quantum interaction length) and the Schwarzschild radius (the gravitational interaction length) rule the astrophysical structures. From this point of view, astrophysical structures are the macroscopic results of average stochastic processes which select their characteristic sizes (e.g. see the book by Roy [18]). Finally, dark matter is not necessary in this framework since its effect (in particular "the missing matter issue") is addressed considering the gravitational interaction depending on scale. This is the viewpoint of several alternative theories of gravity which address the dark energy and dark matter problems without assuming new "dark" ingredients (up to now not detected at fundamental level) but shift the problem to the gravitational sector [19].

### IV. REPRODUCING THE EDDINGTON-WEINBERG RELATION

The above considerations can be used to reproduce some relations connecting quantum and cosmological scales. In particular, the famous Eddington-Weinberg relation brings together the quantum unit of action  $\hbar$  with the radius of the universe. It is interesting to derive this relation within our approach which, as we have seen, is based on such a link. In order to do so, let us start from Eq. (25), specializing it to the case of the universe, that is Eq.(26). Such a relation can be rewritten as

$$R_u \simeq 10 \sqrt{\frac{M_u}{M_p}} \left(\frac{\hbar}{M_p c}\right), \quad (27)$$

where the Compton length has been explicitly given. Writing it in a more convenient form

$$\frac{R_u}{\sqrt{M_u}} = 10 \frac{h}{M_p^{\frac{3}{2}} c}, \quad (28)$$

and using  $\frac{R_u}{\sqrt{M_u}} = \frac{\sqrt{GR_u}}{c}$  from Eq. (19), we get, after substituting and rearranging terms,

$$h = \frac{1}{10} \sqrt{GR_u M_p^3} \quad (29)$$

which fully reproduces Eddington-Weinberg relation (apart from the correcting factor  $\frac{1}{10}$ ). As a consistency check, the value of the radius of the universe can be obtained from Eq. (29). It is

$$R_u \simeq 10^{28} \text{ cm}, \quad (30)$$

in agreement with the corresponding value given in Eq. (26).

## V. RECOVERING THE CURRENT VALUE OF COSMOLOGICAL CONSTANT

Another important quantum-cosmological relation is the Zeldovich relation for cosmological constant. In Ref. [21], Zeldovich, inspired by the Dirac and Eddington works on the large numbers coincidence [1–5], was able to relate the cosmological constant to the mass of the proton. Basically the ratio

$$\frac{R_u}{\lambda_p}, \quad (31)$$

of the universe radius  $R_u$  to the Compton wavelength of the proton  $\lambda_p = \frac{h}{M_p c}$ , the ratio of the age of the universe  $T_u = \frac{R_u}{c}$  to the characteristic time for the light to cross the proton  $T_p = \frac{\lambda}{c}$  are of the order  $10^{40 \div 42}$ . On the other hand, a dimensionless quantity, characterizing the gravitational interaction for the proton, was observed to be almost of the same order of magnitude

$$\frac{\hbar c}{GM_p^2} \simeq 10^{38}. \quad (32)$$

Without entering into the question of variability of the first two ratios with the expansion of the universe or the question of  $G$  variability [21], Dirac strongly believed that the numerical agreement between the ratios given in the above Eqs.(31) and (32) was not accidental but had some deep meaning (Coincidence Principle). Equating Eq.(31) with (32), an interesting formula can be obtained relating again the radius of the universe  $R_u$  to the Compton wavelength of the proton:

$$R_u = \left( \frac{\hbar}{M_p c} \right)^3 \frac{c^3}{G \hbar}. \quad (33)$$

Assuming the presence of cosmological constant  $\Lambda$ , the Coincidence Principle can be reinforced replacing the universe radius  $R_u$  by the quantity  $\Lambda^{-\frac{1}{2}}$  which has the same dimensions. Zeldovich obtained [21]:

$$\Lambda = \left( \frac{\hbar}{G c^3} \right)^2 \left( \frac{M_p c}{\hbar} \right)^6 \quad (34)$$

which is an outstanding expression for the cosmological constant even though it is six orders of magnitude greater than its current value. It is possible to recover immediately the correct numerical value for  $\Lambda$  adopting our approach. In fact, using Eq. (26) for  $R_u$  and

$$\sqrt{n_p^u} = 10^2 \left( \frac{M_{BH}}{M_p} \right)^2 \text{ from Eq.(21), we get}$$

$$R_u = 10^3 \left( \frac{\hbar}{M_p c} \right)^3 \frac{1}{l_{Planck}^2}, \quad (35)$$

where the Planck length  $l_{Planck} = \sqrt{\frac{\hbar G}{c^3}}$  has been adopted. Then, we obtain

$$\begin{aligned} \Lambda &= \frac{1}{10^6} \left( \frac{l_{Planck}^4}{\lambda_p^6} \right) = \\ &= \frac{1}{10^6} \left( \frac{\hbar G}{c^3} \right)^2 \left( \frac{M_p c}{\hbar} \right)^6 \simeq 10^{-60} \text{ cm}^{-2}, \end{aligned} \quad (36)$$

which agrees with the order of magnitude of the observed cosmological constant. In fact, evaluating the corresponding energy density  $\rho_\Lambda = \frac{c^2 \Lambda}{8\pi G}$ , one obtains

$$\rho_\Lambda = \frac{1}{10^6} \left( \frac{GM_p^2}{8\pi \hbar c} \right) \left( \frac{M_p c}{\hbar} \right)^3 \simeq 10^{-29} \frac{g}{\text{cm}^3} \quad (37)$$

There is another conceptually interesting way to write the above equation. It consists in introducing the number of protons in the universe, according to

$$\frac{GM_p^2}{\hbar c} = \left( \frac{M_p}{M_{Planck}} \right)^2 = \frac{10^2}{\sqrt{n_p^u}}, \quad (38)$$

derived from Eq. (21). Substituting into the previous expression, we obtain

$$\rho_\Lambda \simeq \frac{4}{\sqrt{n_p^u}} \left( \frac{M_p}{\frac{4}{3}\pi R_p^3} \right), \quad (39)$$

where  $R_p$  is the radius of the proton according to Eq. (26). Notice that the fluctuation factor  $(\sqrt{n_p^u})^{-1}$  appears in the above equation and it has a relevant role in the statistical hypothesis [15, 16]. We can multiply and divide by the factor  $n_p^u$  obtaining

$$\rho_\Lambda \simeq 4 \left( \frac{M_u}{\frac{4}{3}\pi R_u^3} \right) \simeq 4\rho_m \quad (40)$$

where  $\rho_m$  is the matter density of the universe. In this way, the so-called *Coincidence Problem*, consisting in the fact that the today observed density of dark energy and dark matter are unnaturally comparable in order of magnitude, could be naturally addressed. It is important to stress that also here the scaling relation (33) has been used.

## VI. CONCLUSIONS

In this paper we have shown that the physical sizes of self-gravitating astrophysical structures, roughly described by Eq. (25), are naturally "imprinted" at the very early stages of the Universe, that is at the time when quantum fluctuations play a crucial role. The quantization relation (10) rules, in principle, all the self-gravitating systems up to the whole universe. In other words, such a relation provides a straightforward generalization of the Eddington-Weinberg relation starting from the Dirac quantization. Furthermore, also the Zel'dovich relation between the cosmological constant  $\Lambda$  and the Compton wavelength of the proton can be easily recovered in our approach. In this case, the so-called *Coincidence Problem* is naturally addressed. Finally, being the interactions and the granular components that rule the self-gravitating structures, dark matter is not necessary as further ingredient to build up and stabilize astrophysical systems. A final consideration is in order at this point. The presented results are far to be the definite answer to the problem of connecting quantum to cosmological scales, however they could be useful indications in view to address the problems of large scale structure and dynamics of self-gravitating astrophysical systems.

- 
- [1] P.M.A. Dirac, *Proc. Roy. Soc. A* **133**, 60, (1931).
  - [2] P.A.M. Dirac, *Proc. Roy. Soc. A* **165**, 199, (1938).
  - [3] P.A.M. Dirac, *Nature* **139** 323, (1937).
  - [4] P.A.M. Dirac, *Proc. Roy. Soc. A* **338**, 439, (1974).
  - [5] A.S. Eddington, *Proc. of the Cambridge Philosophical Society*, **27**, 15 (1931).
  - [6] S. Weinberg, *Gravitation and Cosmology*, Wiley, New York (1972).
  - [7] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
  - [8] S. Capozziello, S. De Martino, S. De Siena, F. Illuminati, *Mod. Phys. Lett. A* **15**, 1063 (2000).
  - [9] J.J.Halliwell *Proceedings of the 1990 Jerusalem Winter School on Quantum Cosmology and Baby Universes*, Ed. S.Coleman, J.B.Hartle, T.Piran and S.Weinberg (World Scientific, Singapore, 1991).
  - [10] S. Capozziello, G. Cristofano, M. De Laurentis, *Eur. Phys. J. C* **69**, 293, (2010).
  - [11] R. Kallosh, A. D. Linde, T. Ortín, A. W. Van Proeyen, *Phys. Rev. D* **46**, 5278, (1992).
  - [12] S. Fubini, *Mod.Phys.Lett. A* **6**, 347 (1991).
  - [13] G. 't Hooft, *Nucl. Phys.* **138**, 1 (1978).
  - [14] S. Ferrara, K. Hayakawa, A. Marrani, *Fortsch. Phys.* **56**, 993 (2008).
  - [15] S. Capozziello and S. Funkhouser, *Mod. Phys. Lett. A* **24**, 1121 (2009).
  - [16] S. Capozziello and S. Funkhouser, *Mod. Phys. Lett. A* **24**, 1743 (2009).
  - [17] F. Calogero, *Phys. Lett. A* **228**, 335 (1997).
  - [18] S. Roy, *Statistical Geometry and Applications to Microphysics and Cosmology* Kluwer (1998).
  - [19] S. Capozziello and V. Faraoni, *Beyond Einstein Gravity: A Survey of Gravitational Theories for Cosmology and Astrophysics*, Springer, New York (2011).
  - [20] S. Mendoza, X. Hernandez, J. C. Hidalgo, and T. Bernal, *Mon. Not. R.Astr.* **411**, 226 (2011).
  - [21] Ya. B. Zeldovich, *Pisma ZhETF* **6**, 883 (1967).